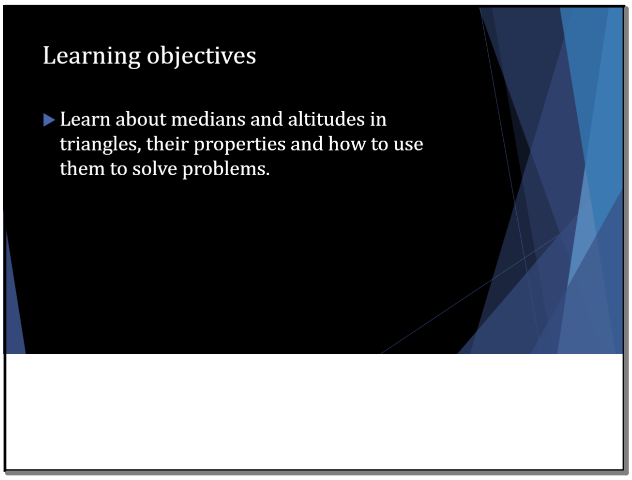
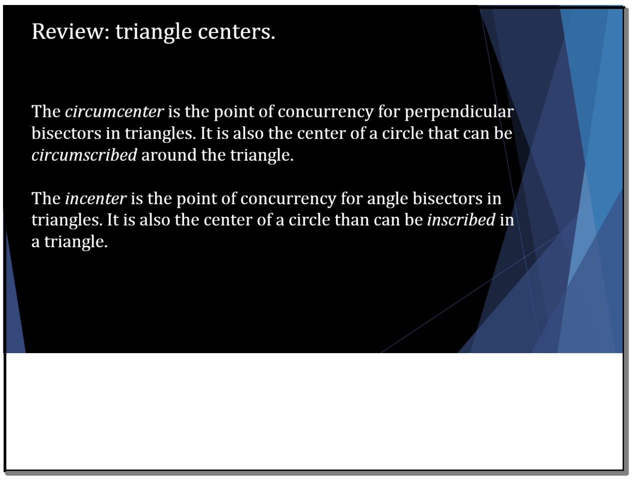
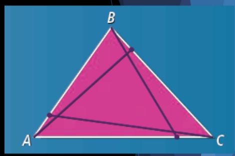

Learning objectives
▶ Learn about medians and altitudes in triangles, their properties and how to use them to solve problems.


Review: triangle centers.
The *circumcenter* is the point of concurrency for perpendicular bisectors in triangles. It is also the center of a circle that can be *circumscribed* around the triangle.
The *incenter* is the point of concurrency for angle bisectors in triangles. It is also the center of a circle than can be *inscribed* in a triangle.

Discovery learning!

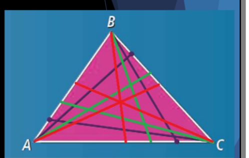
Draw a large *acute scalene* triangle ABC. On each side, mark the point that is 1/5 of the distance from one of the side's endpoints, as shown in the diagram to the right.



Connect each of these points to the opposite vertex. Repeat the process for 1/4 and 1/3. What do you think the result will be for 1/2?

Discovery learning!

The last set of lines (in red) is drawn from the halfway point (the midpoint) to the opposite vertex.

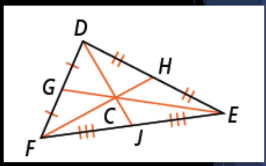


These lines are *medians*: segments that start at a midpoint and end at the opposite vertex.

These concurrent lines intersect in (another) triangle center called the *centroid*.

Theorem 5-8: Concurrency of Medians Theorem

The medians of a triangle are *concurrent* at a point that is 2/3 the distance from each vertex to the midpoint of the opposite side.



$DC = \frac{2}{3} DJ$

$EC = \frac{2}{3} EG$

$FC = \frac{2}{3} FH$

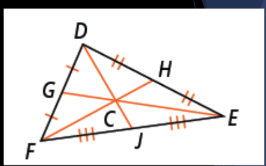
In a triangle, the point of concurrency of the medians is the **centroid of the triangle.** The centroid is always inside the triangle.

Theorem 5-8: Concurrency of Medians Theorem

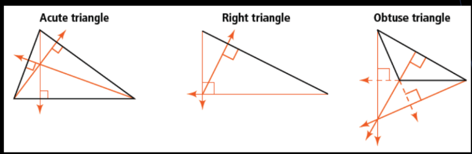
If DJ is 12, what is DC?

If EC is 6, what is EG?

If FC is 4x and FH is 2x – 5 what must x be?

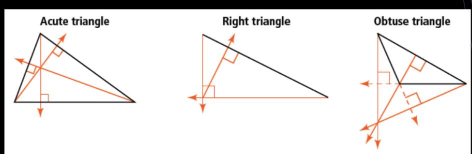


The *altitude* of a triangle is a perpendicular line segment drawn from the vertex to the opposite side. It can be inside, outside or one of the sides of the triangle.



The lines that contain the altitudes of a triangle are concurrent at the **orthocenter of the triangle.** The orthocenter of a triangle can be inside, on, or outside the triangle, just like the altitudes that create it.

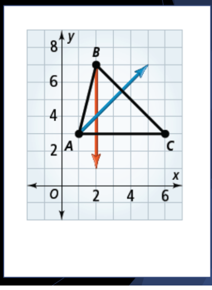
Theorem 5-9: Altitudes are concurrent.



The point of concurrency for altitudes is a triangle center called the *orthocenter.*

Triangle ABC has vertices $A(1, 3)$, $B(2, 7)$, and $C(6, 3)$. What are the coordinates of the orthocenter of triangle ABC?

First, find the equation of the line containing the altitude to AC. Since AC is horizontal, the line containing the altitude to AC is vertical. The line passes through the vertex $B(2, 7)$. The equation of the line is $x = 2$.

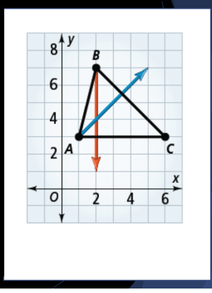


Triangle ABC has vertices $A(1, 3)$, $B(2, 7)$, and $C(6, 3)$. What are the coordinates of the orthocenter of triangle ABC?

Second, find the equation of the line containing the altitude to BC. The slope of the line containing BC is $(3 - 7)/(6 - 2) = -1$.

A perpendicular line has *opposite inverse slope*, so the line containing the altitude to BC has slope 1.

The line passes through the vertex $A(1, 3)$. The equation of the line is $y - 3 = 1(x - 1)$ or $y = x + 2$.

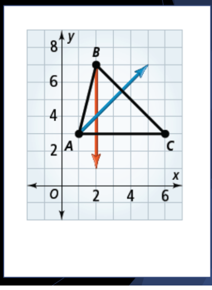


Triangle ABC has vertices $A(1, 3)$, $B(2, 7)$, and $C(6, 3)$. What are the coordinates of the orthocenter of triangle ABC?

Finally, find the orthocenter by solving this system of equations: $x = 2$ and $y = x + 2$

$y = 2 + 2$ Substitute 2 for x in the second equation and simplify to get $y = 4$.

The coordinates of the orthocenter are (2, 4).



Summary